

# Optimum Flight Trajectory Guidance Based on Total Energy Control of Aircraft

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The guidance technique for optimal vertical flight trajectory with a total energy control system (TECS) is discussed. The flight profile is optimized based on the point-mass energy state approximation model of aircraft with direct operating cost as its index function. The guidance law is developed with the total energy control concept used in TECS. To improve the guidance precision, several methods are adopted in the optimization and tracking process, and two preprocessing algorithms, the lead-compensation algorithm and the smooth filtering algorithm, are developed for the ideal optimal trajectory. Satisfactory digital simulation results for a Boeing-707 transport model are finally obtained.

## Nomenclature

$a_n$	= aircraft normal acceleration
$C_f$	= fuel cost coefficient
$C_t$	= time cost coefficient
$D$	= aircraft flight drag
$E$	= total energy state
$\dot{f}$	= fuel rate of aircraft engines
$g$	= gravitational acceleration, 9.806 m/s <sup>2</sup>
$h$	= flight altitude
$K$	= constant control gains in total energy control system
$\dot{L}$	= energy state distribution rate
$s$	= Laplace variable
$T$	= aircraft engine thrust
$t_f$	= ending flight time
$t_0$	= initial flight time
$V$	= flight airspeed
$V_W$	= wind airspeed
$W$	= aircraft weight
$X$	= flight distance
$x, y$	= variables
$\gamma$	= vertical flight path angle
$\lambda$	= cruise cost
$\pi$	= engine thrust setting

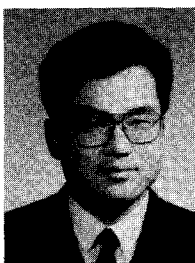
## Subscripts

$c$	= command signals
dn	= descent flight
opt	= optimal profile
up	= climb flight

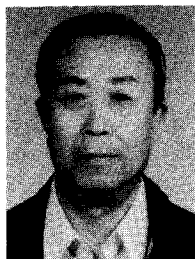
## Introduction

WITH the development of modern control theories and computer science, aircraft flight control systems are becoming more and more integrated. The onboard flight management system (FMS) is one typical integrated system which, comprising the flight performance optimization, flight control, thrust control, guidance computation, as well as navigation management subsystems, can control and cause the aircraft to fly at its optimum performance with minimum cost and fulfill flight missions perfectly and automatically.<sup>1</sup>

In the integrated management and control system of aircraft, guidance law always plays a key role. It plays a part in guiding the aircraft to fly along a specified flight path and greatly influences the overall performance of the whole system. This paper concentrates on the guidance techniques in an



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integrated flight management system that are developed based on the total energy state concept of aircraft.

In recent research, the concept of total energy state has been widely applied in flight performance optimization and integrated flight control problems. To simplify the aircraft dynamic model, a point-mass energy state approximation model was first developed and used in various flight performance optimization problems by Bryson et al.<sup>2</sup> under certain assumptions. A thorough research of the vertical flight trajectory optimization based on this model has been conducted in Refs. 3–6. A set of effective algorithms and software has been developed that can optimize the vertical flight profiles for specified range with direct operating cost (DOC) as an index function. Recently, the total energy concept has also found its application in the integrated flight/propulsion control systems; a new control strategy named total energy control system (TECS) has been developed and studied by Lambregts.<sup>7,8</sup> In this system, coordinated elevator and thrust commands are developed based on point-mass energy principles, which can provide decoupled flight path and speed control for all of the traditional autopilot and autothrottle functions. The flight test evaluated its excellent overall performance with satisfactory testing results.<sup>9</sup>

This paper further studies the optimum trajectory guidance technique based on the total energy concept. Specifically, it studies how the TECS can control aircraft to follow the optimized flight profile with minimum DOC. Beginning with an overview of the trajectory optimization and TECS, the vertical guidance law is first developed, the problems met in the guidance are then discussed, and several methods including the preprocessing algorithms for the ideal optimal profiles are adopted. Finally, the integrated system including the optimization, guidance, and TECS is digitally simulated, and satisfactory results are obtained.

### Trajectory Optimization and Total Energy Control System

As the basis of this paper, let us first give an introduction and overview of the trajectory optimization principle and the integrated flight/propulsion control techniques based on total energy control concepts.

The aircraft energy state is defined as the sum of kinetic and potential energies per unit weight

$$E = h + V^2/2g \quad (1)$$

According to the performances of commercial aircraft flight and under certain assumptions, the point-mass energy state approximation model developed by Bryson et al.<sup>2</sup> describes the aircraft flight dynamics in vertical profile with the following equations:

$$\frac{dE}{dt} = \frac{V(T-D)}{W} \quad (2)$$

$$\frac{dX}{dt} = V \cdot \cos \gamma + V_w \quad (3)$$

Based on this model, in Refs. 3–6 an algorithm is developed for optimizing vertical flight trajectories with a range constraint. The basic assumption made in the algorithm is that the optimum vertical flight paths consist of, at most, three segments, namely, a climb during which energy increases monotonically, a descent during which energy decreases monotonically, and a cruise during which energy and airspeed are constant. This assumption allows energy to be used as the independent variable or timelike variable during the optimization process. The performance index function is taken to be the direct operating cost, which is defined as the weighted sum of fuel cost and time cost<sup>3</sup>:

$$J = \int_{t_0}^{t_f} (C_f \dot{f} + C_t) dt = \int_{t_0}^{t_f} P \cdot dt \quad (4)$$

With the application of the calculus-of-variables method and the Pontryagin's minimum value principle, the trajectory optimization problem is simplified into pointwise extremization problems of the following algebraic functions<sup>3-5</sup>:

$$\lambda = \min_V \frac{P}{V + V_w} \quad (5)$$

$$\begin{aligned} I_{up} &= \min_{\pi_{up}, V_{up}} H_{up} \\ &= \min_{\pi_{up}, V_{up}} \left[ \frac{P - \lambda(V_{up} + V_{wup})}{\dot{E}} \right]_{\dot{E} > 0} \end{aligned} \quad (6)$$

$$\begin{aligned} I_{dn} &= \min_{\pi_{dn}, V_{dn}} H_{dn} \\ &= \min_{\pi_{dn}, V_{dn}} \left[ \frac{P - \lambda(V_{dn} + V_{wdn})}{|\dot{E}|} \right]_{\dot{E} < 0} \end{aligned} \quad (7)$$

where at each energy level during the climb, descent, or at cruise the controls  $\pi$  and  $V$  obey appropriate constraints, and the optimum controls  $\pi$  and  $V$  are obtained as the values that minimize the variable Hamiltonian  $H_{up}$ ,  $H_{dn}$ , or  $\lambda$ . For a detailed description of the algorithm, please refer to Refs. 3–6 and 10.

TECS is an integrated autopilot/autothrottle control system. Its main objective is to achieve decoupled altitude and speed response of the aircraft through coordinated control of elevator and throttles. References 7–9 and 11 give a detailed analysis and discussion of it.

From the point-mass dynamics of aircraft flight, the required thrust can be expressed as follows for the case of small flight path angle (FPA):

$$T = W \cdot [\gamma + (\dot{V}/g)] + D \quad (8)$$

Substituting Eq. (8) for Eq. (2)

$$\frac{\dot{E}}{V} = \gamma + \frac{\dot{V}}{g} = \frac{T-D}{W} \quad (9)$$

Since the variation in the drag is slower than the variation in thrust, the variation of aircraft total energy is directly related to the thrust variation (i.e., throttle variation). Alternatively, it can be stated that the throttles control the rate at which aircraft energy is changed. On the other hand, the variations of elevator mainly generate pitch moment to change FPA; their effects on the total energy are small and could be neglected. Alternatively, the elevator controls the distribution of total energy between potential and kinetic components (i.e., between altitude and airspeed). Hence, the basic principle is obtained that, from an energy management point of view, the thrust should be used to control the total energy and the elevator to control the desired energy distribution between the FPA and acceleration, or between altitude and speed. This forms the basic concept of TECS.

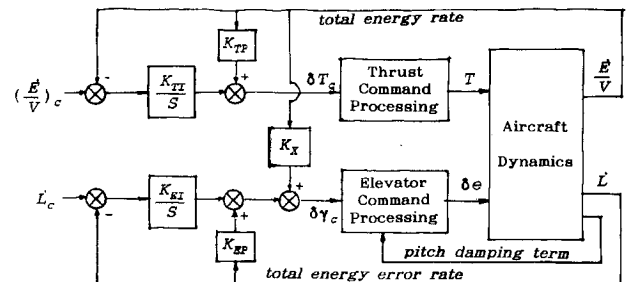


Fig. 1 Basic total energy control systems.

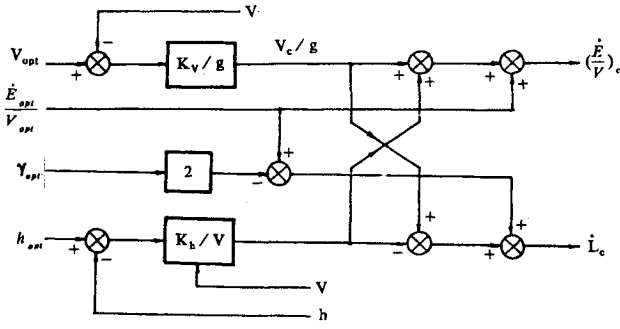


Fig. 2 Block diagram of guidance law.

Define a variable called energy state distribution rate as the difference between kinetic and potential energy rates

$$\dot{L} = (\dot{V}/g) - \gamma \quad (10)$$

and by using a proportional plus integral structure, the control law of TECS has the following forms:

$$\delta T_c = K_{TP} \cdot \frac{\dot{E}}{V} + \frac{K_{TI}}{s} \cdot \left[ \left( \frac{\dot{E}}{V} \right)_c - \frac{\dot{E}}{V} \right] \quad (11)$$

$$\delta \gamma_c = K_{EP} \cdot \dot{L} + \frac{K_{EI}}{s} \cdot (\dot{L}_c - \dot{L}) + K_X \cdot \frac{\dot{E}}{V} \quad (12)$$

where  $K_X$  is the decoupling term that introduces the feedback of total energy rate to the energy distribution channel to realize the decoupling of altitude and speed responses of aircraft flight. The basic structure of TECS is shown in Fig. 1.

For detailed theoretical analysis and system design method of TECS, please refer to Ref. 11, which discusses the design criteria and develops a typical TECS for a Boeing 707 transport aircraft model, including the inner pitch and thrust control loops.

### Guidance Law Development

The main function of guidance law is to compute the control command signals, which are fed to TECS, from the optimal flight trajectory information. As mentioned, the flight trajectory is optimized with the energy state method, so the optimization algorithm can give out four optimal profiles for the flight trajectory: the optimal altitude profile, speed profile, total energy rate profile, and FPA profile. Based on these four optimal profiles, the guidance law is developed as the following formula, with its block diagram shown in Fig. 2.

$$\left( \frac{\dot{E}}{V} \right)_c = \frac{\dot{E}_{opt}}{V_{opt}} + \frac{K_V}{g} (V_{opt} - V) + \frac{K_h}{V} (h_{opt} - h) \quad (13)$$

$$\dot{L}_c = \frac{\dot{E}_{opt}}{V_{opt}} - 2\gamma_{opt} + \frac{K_V}{g} (V_{opt} - V) - \frac{K_h}{V} (h_{opt} - h) \quad (14)$$

Here, the term  $\dot{E}_{opt}/V_{opt} - 2\gamma_{opt}$  equals the optimal energy distribution rate signal.

The distinguishing characteristic of this guidance law is the application of the optimal energy rate and energy distribution rate signals in computing the control command signals for TECS. As known, the energy rate represents the changing rate of both altitude and speed. Hence, there equally exist the derivative signals of optimal altitude and speed profile in the guidance law. This enables it to control aircraft through TECS to fly at the optimal flight profile precisely; digital simulation results given later demonstrate this. For the case of no derivative signals in the guidance computation, i.e., only the latter two terms are used in Eqs. (13) and (14), digital simulations show that there would exist unacceptable tracking errors in both altitude and speed profiles during the climb and descent flight.

### Problems and Measures in Guidance

If the stated guidance law is used directly to relate the optimal flight profile and TECS, it would be found that the whole system cannot work well and there will exist large delays and overshoots relative to the optimal profile in the tracking flight process, because of the large step change and high-frequency vibration component in the optimal FPA and  $\dot{L}$  profile, as shown in Fig. 3. Analyses show that the reasons for the discrepancies could be traced back to the point-mass energy state approximation model used in the trajectory optimization algorithm.

In the point-mass energy state approximation model, a basic assumption was made that the kinetic energy and potential energy could be exchanged at zero time during flight, which means that the changing rates of FPA and flight speed could reach infinity, the FPA and flight acceleration could be exchanged for each other at zero time, i.e., the acceleration of aircraft flight could reach infinity. This makes it possible that there could exist large step changes and high-frequency vibrations in the optimized FPA and energy distribution profile. At the same time, the optimization search range of  $V$  and  $\pi$  in Eqs. (6) and (7) is affected by many factors<sup>3-5</sup> that often severely limit the searching range. For example, when the altitude is under 10,000 ft, the calibrated airspeed of the aircraft is not permitted to surpass 250 kt according to the air traffic control rules. This will bring about a big step change in the optimal FPA and  $\dot{L}$  profile when the altitude reaches 10,000 ft. Because of various restrictions on the searching range for  $V$  and  $\pi$ , the optimality often occurred at the boundary points (minimum or maximum value) of the searching range, which in some cases would make the altitude and speed change alternately. For example, during climb profile optimization, the increase of total energy could result in the climb of altitude in one energy step with speed changing little, whereas in the next energy step, it may result in the speed acceleration with altitude

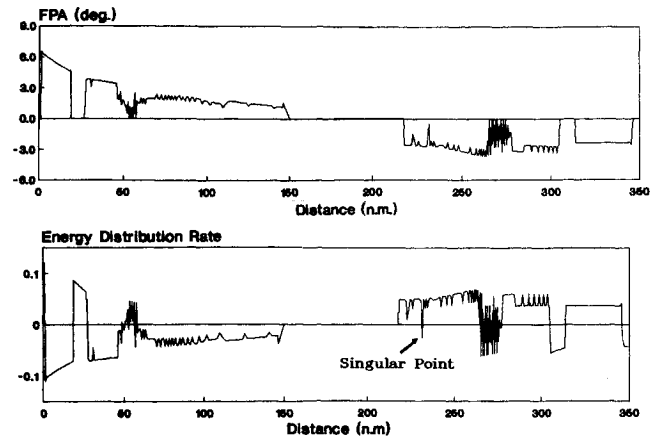
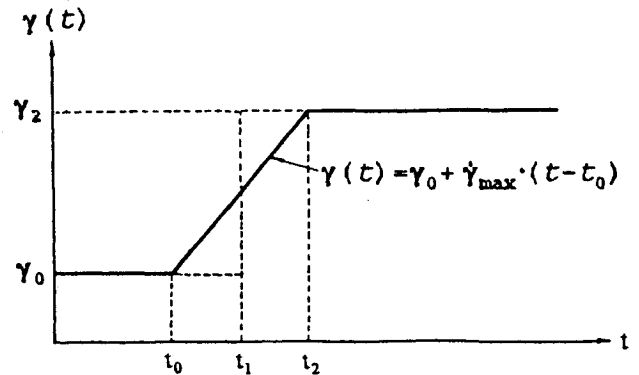
Fig. 3 Ideal optimal FPA and  $\dot{L}$  profile.

Fig. 4 Lead compensation principle.

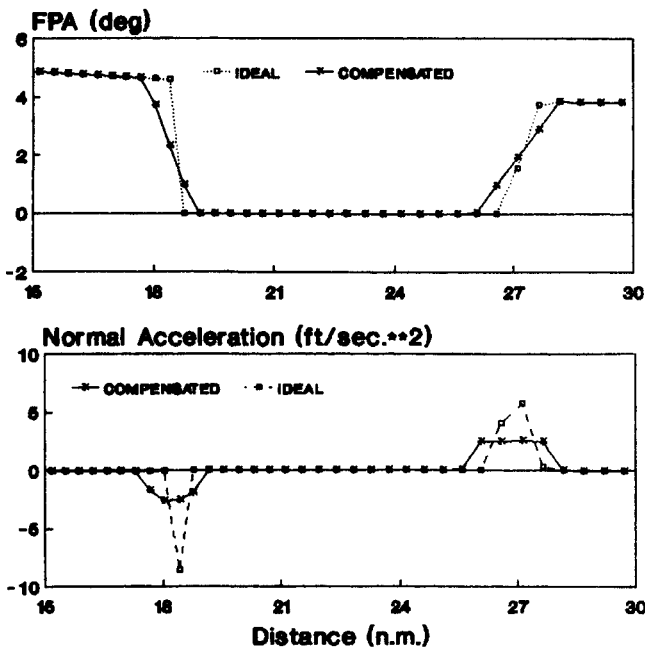


Fig. 5 Lead compensation results.

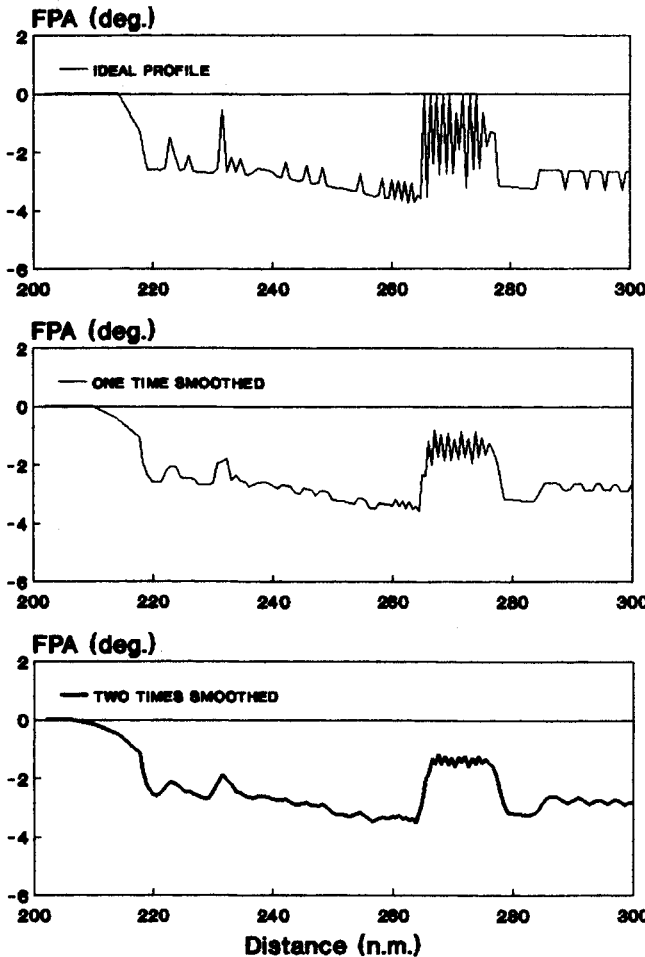


Fig. 6 Smooth filtering of FPA profile.

varying little, and so on. This alternately changing process is illustrated in Fig. 7, which brings out the vibration in FPA and acceleration magnitude in the optimized profiles, as can be seen in Figs. 3 and 6. Figure 3 illustrates the profiles of FPA and energy distribution rate in a typical optimal flight trajectory obtained in Ref. 10, which obviously show the characteristics first discussed. In practical flight, the normal acceleration is limited to 0.1 g for commercial aircraft; hence, this ideal optimal profile cannot be followed directly by actual aircraft.

From the preceding discussion and the digital simulations, we find the following problems for TECS to track the ideal optimal flight profile accurately.

1) In climb profiles, the ideal throttle position is typically on its maximum. Therefore, in the tracking process, the aircraft has no thrust margin to null out the delays of altitude or speed caused by various disturbances. Alternatively, the aircraft has no extra thrust to decrease the amount of delay in total energy, and so there will often exist a constant energy delay in the climb tracking flight.

2) Similar to climb profiles, the ideal thrust setting is typically at idle in descent profiles, and so there will often exist a constant lead error in total energy compared to the ideal descent profile.

3) Because of the limitation on normal acceleration, the aircraft cannot realize the large step changes in FPA and energy distribution rate which are needed in the ideal optimal profile. There will exist a great delay or lead tracking error when this occurs.

4) The high-frequency vibration components and singular points in the ideal optimal profile will lead to vibrations in elevator and normal acceleration and decrease the flight quality of commercial aircraft. Here, the singular point refers to that point which departs from its neighboring points suddenly, as indicated in Fig. 3.

To solve these problems, some adaptive measures are introduced to the trajectory optimization algorithm and aircraft flight simulation, and several preprocessing methods are added to the obtained ideal optimal flight profile. They can be summarized as follows.

1) During the climb profile optimization, the maximum cruise thrust is used as the maximum thrust limitation, instead of the maximum climb thrust previously used. For the JT3D-7 turbofan engine, the maximum cruise thrust is always a little smaller than its maximum climb thrust, and so there exists a small margin thrust for aircraft to track the optimal profile.

2) In the simulation of descent flight, the minimum limitation of thrust is taken as slightly smaller than idle thrust that is used as the minimum value in the optimization algorithm. Actually, the deployment of flaps or flight spoilers will increase the flight drag; this is equivalent to decreasing the thrust (negative thrust). Thus, here we can use a negative thrust to serve the control margin during descent.

3) To improve the continuity and accuracy of optimal profiles, a smaller energy step of 200 ft is used in the optimization algorithm, instead of 500 ft as used previously. This greatly increases the number of points describing the optimal profile and improves its continuity and accuracy.

4) A lead compensation algorithm is developed to smooth the large step changes in FPA profile. A lead time for the FPA command is introduced when a large step change occurs to reduce the changing rate of FPA to its allowable value. The relevant optimal altitude and speed profiles are also revised to adapt to this compensation.

5) A smooth filtering algorithm is finally developed to filter out the high-frequency vibration components and singular

Table 1 Comparison of simulation results

Index	Fuel in climb, lb	Fuel in cruise, lb	Fuel in descent, lb	Fuel in total, lb	Total flight time, s	Total cost, dollar
Ideal profile	8006.25	1677.46	616.25	10299.81	3142.51	1081.2
Simulated profile	8050.19	1751.97	681.82	10483.97	3138.54	1092.2
Relative error, %	0.549	4.44	10.64	1.788	-0.126	1.017

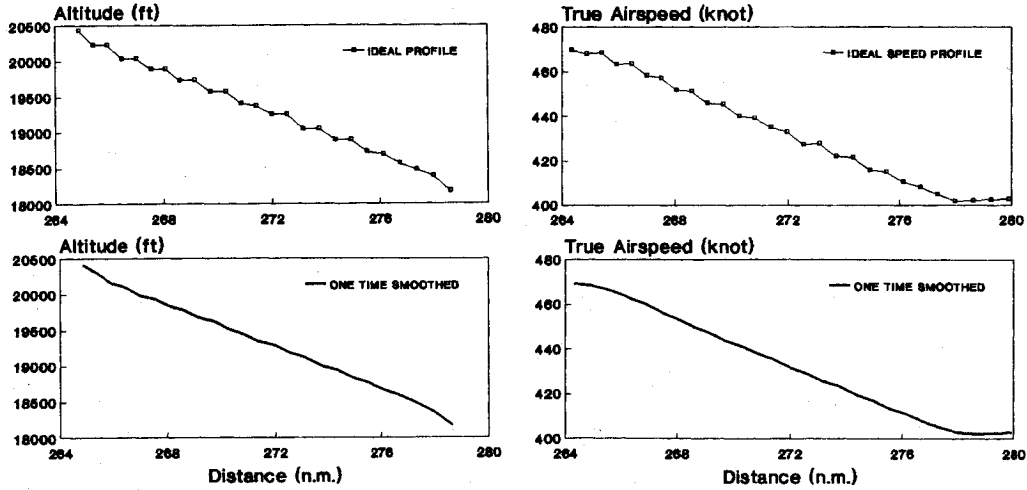
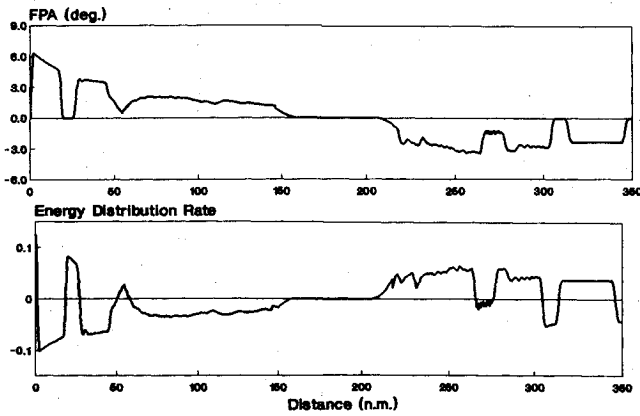


Fig. 7 Smooth filtering of altitude and speed profiles.

Fig. 8 Processed optimal FPA and  $\dot{L}$  profile.

points in optimal flight profiles, making them continuous and steady.

### Preprocessing Algorithms

#### Lead Compensation Algorithm

The basic idea of lead compensation is to use a ramp in FPA to replace the step changes; see Fig. 4. The criteria for this translation is that the amount of resultant altitude change during time  $[t_0, t_2]$  produced by the ramp of FPA should be equal to the amount of change produced by the step change of FPA during the same time period. The lead time of the ramp command is given by the following equation:

$$\tau = t_1 - t_0 = t_2 - t_1 = (\gamma_2 - \gamma_0)/2\dot{\gamma}_{\max} \quad (15)$$

where  $\dot{\gamma}_{\max}$  is the maximum allowable FPA rate, which is determined by the limitation on normal acceleration as in the following equation:

$$\dot{\gamma}_{\max} = \alpha_{n \max}/V \quad (16)$$

where  $\alpha_{n \max}$  is the maximum allowable normal acceleration. For commercial aircraft, it is generally set at  $0.1g$ .

An appropriate logic and algorithm is developed to realize the compensation discussed earlier for the optimal FPA profile.<sup>10</sup> Figure 5 gives the compensation results for a specific portion of the FPA profile in Fig. 3. The relevant normal acceleration profile is also given. Additionally, to adapt to the FPA compensation, the relevant altitude and speed profiles are also revised with the following formulas:

$$h_i = h_{i-1} + V_i \cdot \gamma_i \cdot \Delta t_i \quad (17)$$

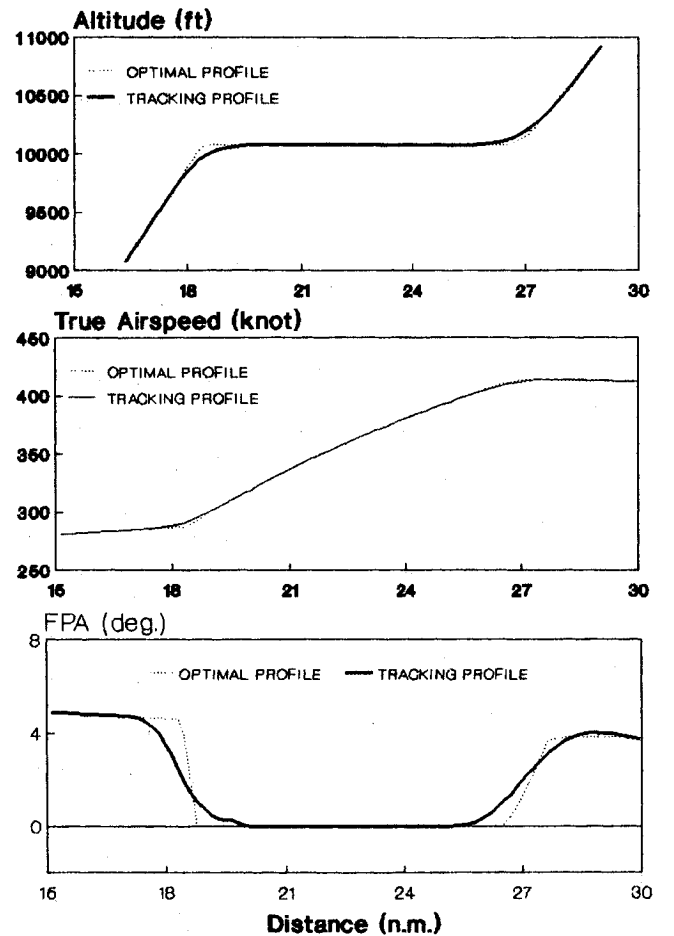


Fig. 9 Simulated tracking results.

$$V_i = \sqrt{2g(E_i - h_i)} \quad (18)$$

where  $\Delta t_i$  is the time period between the  $(i-1)$ th and  $i$ th point, and  $E_i$  is the total energy amount at  $i$ th point.

#### Smooth Filtering Algorithm

The major purpose of this algorithm is to strain the optimal flight profile free from high-frequency vibrations and singular points. An average weighted formula is used here as the filtering algorithm,

$$y(i) = [x(i-1) + x(i) + x(i+1)]/3 \quad i = 2, 3, \dots, n-1 \quad (19)$$

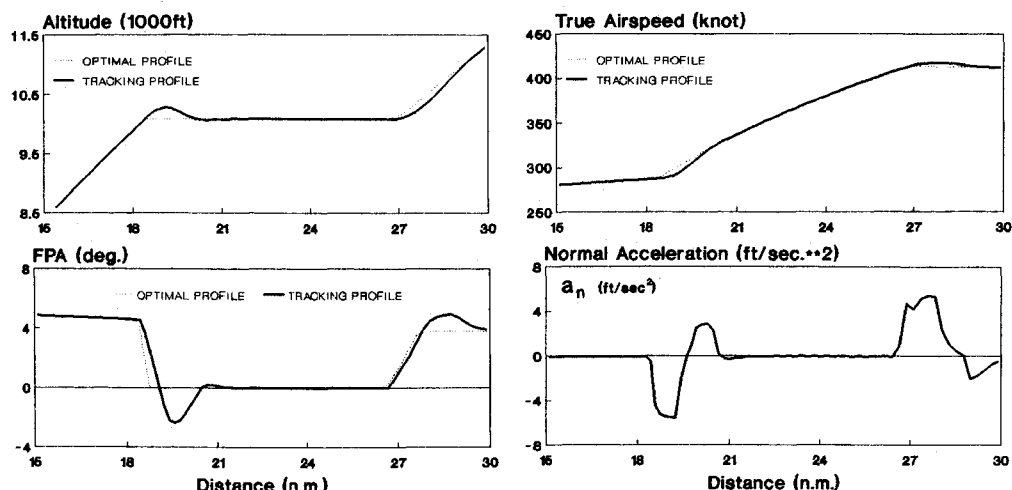


Fig. 10 Tracking results for the ideal optimal profile.

where  $n$  is the number of points of the profile,  $x(i)$  represents the  $i$ th point of the ideal profile, and  $y(i)$  is the resultant profile. Here, only three points are used to compute the central point; increasing the number could smooth the profile better but would change the useful low-frequency information more. For the FPA profile, two times of filtering are used to smooth vibration and decrease the normal acceleration to be within the allowable value (0.1 g). Figure 6 gives the filtering results for a typical portion of the FPA profile. For the altitude and speed profile, only one time of filtering is needed; see Fig. 7.

### Simulation Results

Through the two preprocessing algorithms already discussed, the ideal optimal profile becomes more smooth and continuous, which is suitable for aircraft to track and realize. Figure 8 shows the processed results of the ideal optimal profile shown in Fig. 3. Obviously, it becomes smoother and steadier.

Finally, we use the TECS to track the processed optimal flight profile through the guidance law developed earlier. The simulation results are very satisfactory. The TECS can control the aircraft to track and realize the optimal profile accurately with acceptable elevator and throttle operations, as well as acceptable normal acceleration. Comparing the actual simulated flight profile with the processed optimal profile, it can be found that in a steady climb, cruise, and descent flight, the altitude error is less than 5 ft and speed error less than 0.1 kt; and in the transition flight (climb to cruise, cruise to descent, etc.), the maximum altitude error is less than 30 ft, and speed error less than 2 kt. Table 1 summarizes the simulation results for each flight segment as well as the whole performance index. Figure 9 gives a typical portion of the whole profile simulation result, which is the most difficult portion to be tracked.

For comparison purposes, the simulated profile for the same portion of a flight for TECS to track directly the ideal optimal profile is given in Fig. 10. Obviously, there exists a large delay in airspeed, great overshoots in altitude and FPA, and unacceptable normal acceleration (greater than 0.1 g). This comparison shows the importance of the role of the preprocessing algorithms in the optimum trajectory guidance.

### Conclusions

The total energy state of aircraft in flying is a useful variable in representing the movement of aircraft in airspace. Based on this concept, the performance optimization, integrated flight/propulsion control, and vertical guidance subsystems can be completely integrated into an organic flight and management system.

The basic assumption made in the point-mass energy state approximation model, that its kinetic energy and potential energy could be exchanged instantaneously, is the major reason for the existence of large step changes and high-frequency

vibrations in the ideal optimal flight profiles, which makes it difficult to be tracked by aircraft. But with some preprocessing algorithms, this problem could be solved, and the resultant optimal profile would be easy to realize. Therefore, the energy state method of optimizing flight trajectory is an effective and useful method.

The total energy control concept of aircraft is a good idea for the integrated flight/propulsion (or path/speed) control of aircraft. The TECS can be used to control the aircraft to track and realize its optimum flight trajectory.

The vertical guidance law developed in this paper has several advantages. Derived based on the total energy control concept, it makes it very simple to relate the optimum flight profile with the TECS. The optimal changing rate and distribution rate of the total energy state used in the guidance law actually serve the role of derivative control for the altitude and speed. Thus, it can guide the aircraft through TECS to follow the optimal flight profile accurately.

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